# STATISTICS

## Paper I : Non-Parametric Methods and Regression Analysis

**Note :** Attepmt **all** questions from **Section-A** (Objective type questions), **five** questions from **Section-B** (Short answer type questions) and **three** questions from **Section-C** (Long/Essay type questions).

### Section—A $1 \times 10 = 10$

**1.** Let $X_1, \ldots, X_n$ be a random sample form a $p$-variate normal distribution $N(\underset{\sim}{\mu}, \Sigma)$ and $Y = \sum_{i=1}^{h} X_i$. The distribution of Y is :

(a) $N(n\mu, n\Sigma)$ (b) $N(n\mu, n^2\Sigma)$ (c) $N\left(n\mu, \frac{1}{n}\Sigma\right)$ (d) $N(n\mu, \Sigma)$.

**2.** Let $X \sim N_p(0, \Sigma)$ and P is a $p \times p$ non-singular matrix such that PX $\sim N_p(0, I)$. Then :

(a) P is a identity matrix (b) P is an idempotent matrix

(c) $P'\Sigma P = I$ (d) $P\Sigma P' = I$.

**3.** Let $X_1, \ldots, X_N$ be a random sample from $N_p(\mu, \Sigma)$ and $A = \sum_{i=1}^{N} (X_i - \overline{X})(X_i - \overline{X})'$. The MLE is $\Sigma$ is :

(a) $\frac{1}{N-1}A$ (b) $\frac{1}{N}A$ (c) A (d) $\frac{N}{N-1}A$.

**4.** The marginal pdf of $n^{th}$ order statistics of a random sample of size $n$ is :

(a) $n[F(y_{(n)})]^{n-1} f(y_{(n)})$ (b) $n!(y_{(n)})$

(c) $n \sim \prod_{i=1}^{n} f(y_{(i)})$ (d) $n![F(y_{(n)})]^{n-1} f(y_{(n)})$.

**5.** The joint pdf of all $n$-order statistics is given as :

(a) $n[F(x)]^{n-1}$ (b) $(n-1)[F(x)]^{n-1}$

(c) $n!f(x_{(1)}) \ldots f(x_{(n)})$ (d) None of these.

**6.** Which of the following test is used to test randomness of given sample ?

(a) Sign test (b) Median test (c) Run test (d) Wilcoxon test.

**7.** Ordinary sign test utilizes :

(a) $t$-distribution (b) Poisson-distribution

(c) Binomial distribution (d) Exponential distribution.

**8.** The ordinary least square estimator of $\beta$ in $\underset{\sim}{Y} = A\underset{\sim}{\beta} + \underset{\sim}{e}$ is :

(a) $A^{-1}\underset{\sim}{Y}$ (b) $(A'A)^{-1}\underset{\sim}{Y}$ (c) $(A'A)^{-1}A'\underset{\sim}{Y}$ (d) $A'\underset{\sim}{Y}$.

9. The least square estimator are :

(a) Period estimator (b) Point estimator

(c) Population estimator (d) Popular estimator.

10. Coefficient of determination measures :

(a) The correlation between X and Y (b) The residual sum of square (c) The explained sum of square (d) How well the sample regression fits the data.

**Section—B** $2 \times 5 = 10$

1. Let X ! N (μ, Σ) and let A be a $p \times p$ non-singular matrix. Derive the p.d.f. of AX.

2. Find MLE of the mean vector for multivariate normal distribution.

3. For the exponential distribution $f(x) = e^{-x}, x \geq 0$, find the cumulative distribution function of $n^{th}$ order statistics.

4. Define order statistic.

5. How does a non parametric test differ from a parametric test ?

6. Describe Kolmogrov-Simirnov test.

7. The following data shows the weight (in kg) of 12 pairs of twin brothers. Apply sign test to examine whether the distribution of weight of twin brothers can assumed to be same :

(37·2, 35·7), (43·5, 42·5) (51·8, 50·2), (53·4, 52·9), (32·8, 33·2), (48·4, 47·4), (36·7, 38·7), (47·8, 45·2), (41·4, (39·2), (41·4, 37·6), (43,·3, 38·2), (38·5, (36·7).

8. Describe a linear regression model and state its assumption.

9. Obtain the least square estimates of the parameters in a simple linear regression model.

10. Explain Run test.

**Section—C** $10 \times 3 = 30$

1. Suppose that the random vectors $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ have a joint multivariate normal distribution. Then a necessary and sufficient condition for $\underset{\sim}{X}$ and $\underset{\sim}{Y}$ to be independent is that cov $(\underset{\sim}{X}, \underset{\sim}{Y}) = 0$.

2. Show that the distribution of the sample mean vector $\bar{x}$ computed from a random sample of size $n$ from a normal $N_p$ (μ, Σ) population is $N_p$ (μ, Σ/$n$).

3. Derive the joint pdf of two order statistics $X_{(r)}$ and $X_{(s)}$.

4. Describe in detail (i) sign test, (ii) median test.

5. In a linear regression model :

$$Y_i = \alpha_0 + \alpha_1 X_i + e_i \qquad i = 1, 2, \dots n$$

Derive the test statistic for testing $H_0 : \alpha_1 = 0$.