STATISTICS
Paper I : Statistical Inference

Note : Attempt **all** questions from **Section-A** (Objective type) and **Section-B** (Short answer type) and **three** questions from **Section-C** (Long/Essay type).

Section—A

1. A function of sample values is called :
(a) Parameter (b) Estimator (c) Statistic (d) None of these.

2. A bias of an estimator can be :
(a) Poisitive (b) Negative (b) Positive and Negative (d) Always zero.

3. If $\hat{\mu}_1 = \frac{1}{20}\sum_{i=1}^{20} x_i$ and $\hat{\mu}_2 = \frac{1}{10}\sum_{i=1}^{10} x_i$ then :
(a) $\hat{\mu}_1$ is more efficient than $\hat{\mu}_2$ (b) $\hat{\mu}_2$ is more efficient than $\hat{\mu}_1$
(c) $\hat{\mu}_1$ and $\hat{\mu}_2$ are equally efficient (d) None of these.

4. In testing of hypothesis problem, suppose we reject the null hypothesis when it is true. This is known as :
(a) error of type I (b) error of type II
(c) Correct decision (d) None of these.

5. Whether a test has one sided or two sided critical region, it depends on :
(a) Alternative hypothesis (b) Composite hypothesis
(c) Null hypothesis (d) Simple hypothesis.

6. Let $X \sim N(0, \sigma^2)$. To test $H_0 : \sigma = \sigma_0$ against $H_1 : \sigma = \sigma_1 (> \sigma_0)$, the best critical region is :
(a) $\sum X_1{}^2 < A$ (b) $\sum X_1{}^2 \geq B$
(c) $\sum (x_1 - \bar{x})^2 < A$ (b) $\sum (x_1 - \bar{x})^2 \geq B$.

7. χ^2-test is used for : $H_0 = \sigma^2 = \sigma_0{}^2$
(a) Testing (b) Testing independence of two attributes
(c) Testing the goodness of fit (d) All of the above.

8. The m... difference between 9 paired observations is 15·0 and the standard deviati... of differences is 5·0. The value of statistic t is :
(a) 27 (b) 9 (c) 3 (d) zero.

9. In likelihood Ratio (L-R) tests, under certain conditions, $-2\log_e \lambda$ has distributed as :
(a) Normal (b) Chi-square (c) Student's t (d) Snedecor's F.

10. For a fixed confidence coefficient, the confidence interval with shortest length is known as :
(a) Central confidence interval (b) Shortest confidence interval
(c) Best confidence interval (d) All of the above.

Section—B

1. Explain unbiasedness and consistency. **Or**

Obtain the m.l.e. of θ for the following distribution :

$$f(x, \theta) = \frac{1}{\theta}, 0 < x < \theta, \theta > 0.$$

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2. Differentiate between :

(i) Simple and Composite hypothesis

(ii) Critical and Acceptance region. **Or**

Define Cramer-Rao inequality with regularity conditions.

3. From the given probability distribution :

$$f(x, \theta) = \frac{1}{\theta}, \qquad 0 \le x \le \theta, \theta > 0$$
$$= 0, \qquad \text{otherwise.}$$

one observation x is taken to test hypothesis $H_0 : \theta = 1$, against $H_1 : \theta = 2$, obtain the size of the type I error for the critical region $0{\cdot}8 \le x \le 1{\cdot}3$. **Or**

Define most powerful and uniformly most powerful test.

4. A random sample of 10 observations, shows a mean of 4·13 with a standard deviation of 0·189. Test the hypothesis $H_0 : \mu = 4{\cdot}0$. **Or**

Define f-test and discuss its uses.

5. A random sample of 500 members is found to have a mean of 3·4 cms. Could it come from a large population with $\mu = 3{\cdot}25$. and $\sigma = 2{\cdot}61$ cms. **Or**

Describe the method of Interval estimation.

Section—C $10 \times 3 = 30$

1. (a) Explain the method of maximum likelihood for estimating the parameter of a population. Also write the properties of m.l.e.

(b) Show that $\bar{x} - 1$ is an unbiased estimator of θ, for the probability density function : $f(x, \theta) = e^{\theta - x}, \theta < x < \infty$.

2. (a) Estimate the parameters μ and σ^2 by the method of moments in the sampling from normal population.

(b) Given, $f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}, 0 \le x \le \infty, \theta > 0$. The null hypothesis $H_0 : \theta = 2$ is rejected and $H_1 : \theta = 3$ is accepted if an observation selected at random takes the value 12 or more. Write the critical region and find the size of the two types of errors.

3. (a) State the prove Neyman-Pearson lemma.

(b) Find the best critical region (BCR) for a random sample of size n drawn from Poisson distribution $f(x, \lambda)$ to test $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (> \lambda_0)$.

4. (a) In a simple sample of 600 men from a certain large city, 450 are found to be smokers. In another sample of 900 from the other city, 480 are smokers. Do the data indicate that the cities are significantly different with respect to prevalence of smoking among men ?

(b) Two random samples drawn from normal population are given below :

Sample A : 20, 16, 26, 27, 23, 22, 18, 24, 25, 19

Sample B : 27, 33, 42, 42, 35, 32, 34, 38, 30, 24, 34.

Test the hypothesis :

Given : $H_0 : \sigma_1^2 = \sigma_2^2$, $F_{0{\cdot}05}(9, 10) = 3{\cdot}05$, $F_{0{\cdot}05}(10, 9) = 3{\cdot}14$.

5. (a) Discuss the general method of construction of likelihood Ratio (L-R) test.

(b) Show that the likelihood ratio test for testing the mean of a normal population is the usual t-test.