STATISTICS

Paper I : Statistical Inference

Note : Attempt **all** questions from **Section-A** (**Objective type**), **five** questions from **Section-B** (Short answer type) and **three** questions from **Section-C** (Long/Essay type questions).

Section—A

1. Bias of a estimator can be :
(a) positive (b) negative
(c) either positive or

2. If t is a consistent estimator of θ, then :
(a) t is also a consistent estimator of θ^2 (b) t^2 is also a consistent estimator of θ (c) t^2 is also a consistent estimator of θ^2 (d) None of the above.

3. Crammer Rao inequality is valid in case of :
(a) continuous variable (b) discrete variable
(c) both (a) and (b) (d) neither (a) nor (b).

4. For testing equality of two variance, the following test is used :
(a) t-test (b) χ^2-test (c) f-test (d) None of above.

5. The maximum likelihood estimators are necessarily :
(a) unbiased (b) sufficient (c) most efficient (d) unique.

6. Method of minimum chi-square for the estimation of parameter utilises :
(a) chi-square distribution function (b) Pearson's chi-square statistic
(c) contingency table (d) All the above.

7. The credit of inventing the method of moments for estimating the parameters goes to :
(a) R.A. Fisher (b) J. Neyman (c) Laplace (d) Karl Pearson.

8. Power of a test is related to :
(a) size of the test (b) type II error
(c) both types of error (d) confidence coefficient.

9. Test of hypothesis $H_0 : \mu = 70$ against $H_1 : \mu > 70$ leads to :
(a) one side left tailed test (b) one side right tail test
(c) two tailed test (d) none of the above.

10. How many types of error in taking a decision about H_0 ?
(a) three type (b) one type (c) two type (d) zero type.

Section—B

1. If T is a consistent estimator of θ, then $\sqrt{T}$ is also consistent estimator of $\sqrt{\theta}$ prove it.

Or

Define method of maximum likelihood estimator.

2. From the given probability distribution :

$$f(x, \theta) = \frac{1}{\theta} \quad 0 \le x \le \theta$$
$$= 0 \quad \text{otherwise}$$

one observation x is taken to test the hypothesis $H_1 : \theta = 2$ against $H_1 : \theta = 2$, obtain the size of the 1st error of the CR $0{\cdot}8 \le x \le 1{\cdot}3$. **Or**

Define the level of significance, critical region, acceptance region.

3. Define completely paired t-test. **Or**

The nine items of a sample have the following values :

Values : 45, 47, 50, 52, 48, 47, 49, 53, 51

test the hypothesis that the mean of population is 47·5.

[Given $t_{.05/2} = 2{\cdot}306$ for 8 degrees for freedom]

4. Define interval estimation. **Or**

Define likelihood ratio test.

5. Obtain the MLE of θ for the following distribution :

$$f(n) = \frac{1}{\theta}, 0 < x < \theta, \theta > 0$$ **Or**

Define power of the test, most powerful test and uniformly most powerful test.

Section—C

1. (a) What are the Fisher's criterion for a good estimator ? Explain them.

(b) Let x_1, x_2, x_3 be a random sample of size $n = 3$ taken from a normal population with mean μ and variance σ^2. Let $\hat{\mu}_1 = \frac{x_1 + 2x_2 + 3x_3}{6}$, $\hat{\mu}_2 = \frac{x_1 + x_2 + x_3}{3}$ be the two estimates of μ.

(i) Are $\hat{\mu}_1$ and $\hat{\mu}_2$ unbiased. (ii) Compare the efficiency of $\hat{\mu}_1$ and $\hat{\mu}_2$.

2. (a) State and prove Crammer Rao inequality.

(b) Estimate and population variance θ in normal distribution and show that $\frac{1}{n}\sum x_i^2$ is a maximum variance unbiased estimator and its variance is $\frac{2\theta^2}{n}$.

3. (a) State and Prove Neman-Pearson Lemma.

(b) Find the best critical region for a random sample of size n from Poisson distribution $f(x, \lambda)$ for test $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1$.

4. (a) A coin is tossed 400 times and head occurred 216 times. Test whether the coin is fair. [7 tab at 5% level of significance is 1·96]

(b) Two random samples drawn from normal population are given below :

Sample A :	20	16	26	27	23	22	18	24	25	19	
Sample B :	27	33	42	42	35	32	34	38	30	24	34

Test the hypothesis $H_0 : \sigma_1^2 = \sigma_2^2$

$$\begin{bmatrix} F(9, 10)(0{\cdot}05) = 3{\cdot}05 \\ F(10, 9)(0{\cdot}05) = 3{\cdot}14 \end{bmatrix}$$

5. (a) Show that in normal population sample mean is more efficiency than sample median.

(b) Let $x_1, x_2, \ldots x_n$ be random observations on a Bernolli variable n takes value 1 with probability θ and value 0 with probability $(1 - \theta)$, show that $\frac{T(T-1)}{n(n-1)}$ is an unbiased estimator of θ^2 where $T = \sum_{i=1}^{n} x_i$.