Paper III : Mechanics

M. M. : B.A. 25/B.Sc. : 50

Note : Attempt any **five** questions from **Section-A** and any **three** questions from **Section-B.**

Section—A

B.Sc. $4 \times 5 = 20$/**B.A.** $2 \times 5 = 10$

1. The velocities of a particle along and perpendicular to the radius from a fixed origin are 'λr' and '$\mu\theta$', find radial and transverse accelerations.

2. A particle moving in straight line with S.H.M. has velocities 'v_1' and 'v_2' when it's distance from the centre are x_1 and x_2. Show that the periodic time of the motion is given by :

$$2\pi \sqrt{\frac{(x_1^2 - x_2^2)}{(v_2^2 - v_1^2)}}$$

3. Find the central force under which a particle describes the lemniscate $r^2 = a^2 \cos 2\theta$.

4. For a common catenary, prove that : $S = c \tan \psi$.

5. Find the centre of gravity of thin uniform hemispherical shell.

6. State the principle of virtual work. Find the work done by the tension in a virtual extension of a string.

7. A heavy uniform rod rests with one end against a smooth vertical wall and with a point in it's length resting on a smooth peg. Find the position of equlibrium and show that it is unstable.

Section—B

B.Sc. $10 \times 3 = 30$/**B.A.** $5 \times 3 = 15$

1. A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest. Prove that the time occupied in the falling down the first half of the vertical height is equal to the time of falling down the second half.

2. An uniform chain of length 'l' which can just bear a tension of n times its weight, is stretched between two points in the same horizontal line, show that the least possible sag in the middle is $l\{n - \sqrt{n^2 - 1/4}\}$.

3. Two light elastic strings are fastened to a particle of mass 'm' and their other ends to fixed points so that the strings are taut. The modulus of each is λ, the tension T and lengths 'a' and 'b'. Show that the period of an oscillation along the line of the strings is

$$2\pi \left[\frac{mab}{(T + \lambda)(a + b)}\right]^{1/2}$$

4. In a Central Orbit the force is $\mu u^3 (3 + 2a^2u^2)$; If the particle be projected at a distance 'a', with a velocity $\sqrt{(5\mu/a^2)}$ in a direction making an angle $\tan^{-1}(1/2)$ with the radius vector, show that the equation to the path is $r = a \tan \theta$.

5. (a) Find the equation to the null plane of a given point (a, b, c) referred to any co-ordinate axes OX, OY, OZ.

(b) Forces x, y, z act along three straight line $y = b, z = -c$; $x = -a, z = c$ and $x = a, y = -b$, show that they will have a single resultant if $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$.