Paper III : Mechanics

Section—A $1 \times 10 = 10/0{\cdot}5 \times 10 = 5$

1. The C.G. of a semi-circular arc of radius a lies at the distance from its centre :

(a) $\frac{2a}{\pi}$ (b) $\frac{\pi}{2a}$ (c) $\frac{a}{2\pi}$ (d) None of these.

2. If T is the time period of a S.H.M. of amplitude a and intensity of force μ], then :

(a) $T = \frac{2\pi}{\sqrt{\mu}}$ (b) $T = \frac{\pi}{2\sqrt{\mu}}$ (c) $\frac{\sqrt{\mu}}{2\pi}$ (d) None of these.

3. For the catenary $y = c\cos h(x/c)$:

(a) $x = c\log\left(\frac{y+s}{2}\right)$ (b) $x = c\tan\psi$

(c) $s = c\cos\psi$ (d) None of these.

4. With usual notations, the transverse component of velocity at time t is :

(a) $r\frac{d\theta}{dt}$ (b) $r\frac{dv}{dt}$ (c) $r\frac{d\theta}{ds}$ (d) None of these.

5. The virtual work done by the reaction of any smooth surface with which the body is in contact is :

(a) Zero (b) Positive (c) Negative (d) None of these.

6. A particle slides down the outside of a smooth vertical circle of radius a starting from rest at the highest point. It will leave the circle after descending vertically a distance equal to :

(a) $\frac{2a}{3}$ (b) $\frac{3a}{2}$ (c) $\frac{a}{4}$ (d) None of these.

7. The condition of equilibrium of a system of forces is :

(a) $R = 0, G \neq 0$ (b) $R \neq 0, G = 0$

(c) $R \neq 0, G \neq 0$ (d) None of these.

8. By increasing the velocity of any body in any medium, the resistance of the medium :

(a) Decreases (b) Remains the same

(c) Becomes zero (d) None of these.

9. A hemi-sphere rests in equilibrium on a sphere of equal readius a. If the curved surface of the hemi-sphere rests on the sphere, the equilibrium is :

(a) Stable (b) Neither stable nor unstable

(c) Unstable (d) None of these.

10. In a central orbit at an apse :

(a) $\frac{1}{r}$ (b) $p = r^2$ (c) $p = 2r$ (d) None of these.

Section—B $3 \times 5 = 15/2 \times 5 = 10$

1. Find the C.G. of the arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ lying in the first qudrant.

Or

The law of motion in a straight line is given by $s = \frac{1}{2} vt$, prove that the acceleration is constant.

2. With usual notations, for a common catenary obtain the formula :

$$x = c \log (\sec \psi + \tan \psi)$$

Or

The velocities of a particle along and perpendicular to the radius vector are λr and $\mu\theta$. Find the equation of the path.

3. Explain the Principle of Virtual work.

Or

A heavy particle is tied to one end of a light inextensible string whose other end is attached to a fixed point. It is projected horizontally with a given velocity u from its vertical position of equilibrium. Obtain equations of motion of the particle along the tangent and normal.

4. A hemisphere rests in equilibrium on a sphere of equal radius, show that the equilibrium is unstable when the curved surface of the hemisphere rests on the sphere.

Or

A particle is falling from rest under gravity in a resisting medium whose resistance varies as the square of the velocity. Obtain the velocity of the particle at any position.

5. Forces P, Q, R act along three straight lines.

$$y = b, z = -c; z = c, x = -a \text{ and } y = a, y = -b.$$

Show that they will have a single resultant, if :

$$\frac{a}{P} + \frac{b}{Q} + \frac{c}{R} = 0$$

Or

A particle describes the curve $r^n = a^n \cos n\theta$ under a force to the pole. Find the law of force.

Section—C $15 \times 3 = 45/6 \times 2 = 12/7 \times 1 = 7$

1. A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex downwards. Discuss the motion.

2. A particle moves in a plane with acceleration which is always directed to a fixed point O in the plane. Obtain differential equation of the path in polar form and pedal form.

3. The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is

$$\mu \log \left[\frac{1 + \sqrt{(1+\mu)^2}}{\mu} \right]$$

where μ is the coefficient of friction.

4. A heavy particle moves in a smooth sphere; if the velocity be that due to the level of the centre, prove that reaction of the surface will vary as the depth below the centre.

5. Find the condition that the straight line

$$\frac{x-f}{l} = \frac{y-g}{m} = \frac{z-h}{n}$$

may be a null line for the system of forces $(x, y, z; L, M, N)$.