MATHEMATICS

Paper I : Linear Algebra and Matrices

Note : Attempt **all** questions from **Section-A** (Objective type questions) and **Section-B** (Short answer type questions) and **three** questions from **Section-C** (Long/Essay type questions).

Section—A $0{\cdot}8 \times 10 = 8,\ 1 \times 10 = 10$

1. If U(F) and V(F) are two vector spaces and T is a linear transformation from U to V, then the range of T is a subspace of :

(a) V (B) U (c) $U \cup V$ (d) $U \cap V$.

2. The set of Vectors $\{\alpha, \beta\}$ is linearly independent if :

(a) $a\alpha + b\beta = 0 \Rightarrow a = 0, b \neq 0$ (b) $a\alpha + b\beta = 0 \Rightarrow a \neq 0, b = 0$

(c) $a\alpha + b\beta = 0 \Rightarrow a = 0, b = 0$ (d) $a\alpha + b\beta = 0 \Rightarrow a \neq 0, b \neq 0$.

3. If V is a finite dimensional vector space, then number of elments in any two bases of V is :

(a) Even (b) Odd (c) Equal (d) None of these.

4. Let V be a vector space over the field F, then $T : V \rightarrow F$ is a linear functional if :

(a) T is one-one (b) T is onto

(c) $T(c\alpha + \beta) = cT(\alpha) + T(\beta)$ (d) T is one-one and onto.

5. Let W be a subspaces of a finite dimensional vector space V, then :

(a) $W^0 \approx \frac{V^*}{W^0}$ (b) $W^0 \approx \frac{V^*}{W^*}$ (c) $W^* \approx \frac{V^*}{W^0}$ (d) None of these.

6. In a inner product space V(F), $\|\alpha + \beta\| \leq$

(a) $\|\alpha\| \cdot \|\beta\|$ (b) $\|\alpha\| + \|\beta\|$

(c) $\|\alpha\| - \|\beta\|$ (d) None of these.

7. The Rank of Matrix $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$ is :

(a) 1 (b) 3 (c) 2 (d) None of these.

8. The following system of equations is consistent if :

(a) $a + b - c = 0$ (b) $a - b + c = 10$ (d) $a + b + c \neq 0$ (d) $a + b + c = 0$.

9. The eigen value of a Hermitian Matrix are :

(a) all zero (b) all imaginary

(c) all real (d) None of these.

10. Atleast one eigen value of every singular matrix is :

(a) 1 (b) -1 (c) 0 (d) None of these.

Section—B $2 \times 5 = 10$

1. Define a vector space and give one example of a finite vector space.

Or

Find the rank of Matrix $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$.

2. Prove that the vectors (1, 1, 0); (3, 1, 3) and (5, 3, 3) are linearly dependent.

Or

Show that the set of unit vectors {(1, 0, 0), (0, 1, 0), (0, 0, 1) is a basis of $V_3(F)$.

3. If W_1 and W_2 are two subspaces of a vector space V over the field F, then the set $W_1 + W_2 = \{x + y : x \in W_1, y \in W_2\}$ is also a subspace of V.

Or

Define Quotient space with example.

4. If $T : V \to V'$ is an isomorphism, then T(B) is a basis for V' if B is a basis of V.

Or

Define the Sylvester law of nullity.

5. Solve the equations by matrix

$$\begin{aligned} x + 2y + z &= 4 \\ x - y + z &= 5 \\ 2x + 3y - z &= 1 \end{aligned}$$

Or

The characteristic roots of a Skew-Hermitian Matrix are either pure imaginary or zero.

Section—C $5 \times 3 = 15/15 \times 3 = 45$

1. If w_1 and w_2 are two subspaces of a finite dimensional vector space V, then :

$$\dim(w_1 + w_2) = \dim w_1 + \dim w_2 - \dim(w_1 \cap w_2).$$

2. If V and V' are two finite dimensional vector spaces over the same field. F, then $V \approx V' \Leftrightarrow \dim V = \text{div } V'$.

3. (a) If (f_1, f_2, f_3) be the dual basis of the basis $B = \{(1, -2, 3), (1, -1, 1), (2, -4, 7)\}$ of $V_3(R)$ then determine.

(b) Let W be a subspace of a inner product space V. If $\{x_1, x_2, \dots x_m\}$ is a basis for W, then show that :

$$y \in W^i \Leftrightarrow (y, x_1) = 0, \forall\, i = 1, \dots m$$

4. (a) Find out for what value of λ, the equations :

$$\begin{aligned} x + y + z &= 1 \\ x + 2y + 4z &= \lambda \\ x + 4y + 10z &= \lambda^2 \end{aligned}$$

have a solution and solve completely in each case.

(b) Find the eigen values for the matrix $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$

5. Verify Caley-Hamilton theorem for the matirix

$$\begin{bmatrix} 1 & h & 0 \\ h & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and find A^{-1}