Paper II : Differential Equations and Integral Transforms

M. M. : B.A. 25/B.Sc. : 50

Note : Attempt any **five** questions from **Section-A** and any **three** questions from **Section-B**.

Section—A

B.Sc. $4 \times 5 = 20$/B.A. $2 \times 5 = 10$

1. Solve : $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

2. Solve : $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$.

3. Solve : $p = \tan(px - y)$.

4. Find the equation of orthogonal trajectories of the curve $xy = a^2$.

5. Form partial differential equations by eliminating arbitrary constants from the following equation : $z = a(x + y) + b$.

6. Prove that : $L\left\{\frac{1}{\sqrt{\pi t}}\right\} = \frac{1}{\sqrt{p}}$.

7. Find the Fourier Transform of $f(x)$ defined by :

$$f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$$

Section—B

B.Sc. $10 \times 3 = 30$/B.A. $5 \times 3 = 15$

1. Solve : $(x + 1)^2 \frac{d^2y}{dx^2} - 3(x + 1)\frac{dy}{dx} + 4y = x^2$.

2. Solve : $x^2 \frac{d^2y}{dx^2} - 2x(1 + x)\frac{dy}{dx} + 2(1 + x)y = x^3$.

3. (a) Solve : $p - q = \frac{z}{x + y}$. (b) Solve : $p + q = px + qy$.

4. (a) Find the Finite Fourier cosine transform of $f(x) = x$ where $0 < x < \pi$. (b) Solve : $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny$.

5. (a) Find : $L^{-1}\left\{\frac{1}{(p-1)^5 (p+2)}\right\}$.

(b) Apply Laplace transform to solve :

$\frac{d^2y}{dx^2} + y = 6\cos 2t$, $y = 3$ and $\frac{dy}{dt} = 1$, when $t = 0$.