

Paper II : Differential Equations and Integral Transforms**Section—A** **$1 \times 10 = 10$**

Choose the correct answer.

1. The order of the differential equation :

$$\frac{d^4y}{dx^4} - 3\left(\frac{d^3y}{dx^3}\right)^2 + 4\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0 \text{ is :}$$

- (a) 3 (b) 6 (c) 4 (d) 2.

2. The integrating factor of $\frac{dy}{dx} + y \cot x = 2 \cos x$ is :

- (a)
- $\sin x$
- (b)
- $\log \sin x$
- (c)
- $\cot x$
- (d)
- $\cos x$
- .

3. Solution of the differential equation $p^2 - 8p + 15 = 0$ is :

- (a)
- $p = 5, p = 3$
- (b)
- $(y - 5x - c)(y - 3x - c) = 0$
-
- (c)
- $(y + 5x)(y + 3x + c) = 0$
- (d) None of these.

4. The P.I. of the differential equation $(D^2 + 1)y = \cos x$ is :

- (a)
- $\frac{1}{2} \sin x$
- (b)
- $-\frac{x}{2} \sin x$
- (c)
- $\frac{x}{2} \sin x$
- (d)
- $\frac{x}{2} \cos x$
- .

5. The equation $Pp + Qq = R$ is known as :

- (c) Charpit's equation (b) Lagrange's equation
-
- (c) Bernoulli's equation (d) Clairaut's equation.

6. The value of the integral $\int_{-1}^1 [P_n(x)]^2 dx$:

- (a)
- $\frac{1}{2n+1}$
- (b)
- $\frac{1}{n}$
- (c)
- $\frac{2}{2n+1}$
- (d) 0.

7. $\sqrt{\frac{\pi}{2}} J_{1/2}(x)$ equals :

- (a)
- $\sqrt{\frac{\sin^2 x}{x}}$
- (b)
- $\sqrt{\frac{\cos x}{x}}$
- (c)
- $\sqrt{\frac{\cos^2 x}{x}}$
- (d)
- $\sqrt{\frac{\sin x}{x}}$
- .

8. The Laplace transform of $\cos h$ at is :

- (a)
- $\frac{p}{p^2 - a^2}$
- (b)
- $\frac{p}{p^2 + a^2}$
- (c)
- $\frac{1}{p^2 - a^2}$
- (d)
- $\frac{1}{p^2 + a^2}$
- .

9. The value of $L^{-1} \left\{ \frac{1}{(p-1)^2} \right\}$ is :

- (a)
- $t^2 e^t$
- (b)
- $\frac{t^2 e^t}{2}$
- (c)
- $\frac{te^t}{2}$
- (d) None of these.

10. The Fourier sine transform of e^{-x} is :

- (a)
- $\frac{1}{1+p^2}$
- (b)
- $\frac{p}{1+p^2}$
- (c)
- $\frac{1}{p^2}$
- (d)
- e^{-p}
- .

Section—B **$5 \times 5 = 25$**

1. Solve the differential equation :

$$\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$$

Or

$$\text{Solve } \cos^2 x \left(\frac{dy}{dx} \right) + y = \tan x.$$

2. Find general and singular solution of $y = px + \frac{q}{p}$.

Or

Solve $(D^2 + D - 6)y = x$.

3. Show that $P_n(1) = 1$ and $P_n(-1) = (-1)^n$.

Or

Prove $J_{-1/2}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

4. Find $L\{t^2 \sin at\}$.

Or

Find $L^{-1}\left\{\frac{3p-2}{p^2-4p+20}\right\}$

5. Find the Fourier cosine transform of e^{-x^2} .

Or

Find the Fourier sine transform of $f(x) = 2x$, $0 < x < 4$.

Section—C

 $10 \times 3 = 30$

1. Solve $(1 + y^2) dx = (\tan^{-1} y - x) dy$.

2. Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x) \frac{dy}{dx} + y = 4 \cos \log(1+x)$.

3. Solve $r - t \cos^2 x + p \tan x = 0$.

4. Show that $L^{-1}\left\{\frac{p}{p^4+p^2+1}\right\} = \frac{2}{\sqrt{3}} \sin h \frac{t}{2} \sin \frac{\sqrt{3}}{2} t$.

5. Find the Fourier series for the function $f(x) = x + x^2$, $-\pi < x < \pi$.

Hence show that :

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots$$