Paper II : Differential Equations and Integral Transforms

Section—A $0{\cdot}75 \times 10 = 7{\cdot}5/1 \times 10 = 10$

1. The order of the differential equation $x\left(\frac{dy}{dx}\right)^3 + x^2y = \log x + A$ is :

(a) 0 (b) 1 (c) 2 (d) 3.

2. The degree of the differential equation $\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 2\frac{d^2y}{dx^2}$ is :

(a) 0 (b) 1 (c) 2 (d) 3.

3. The differential equation of the form $y = px + f(p)$ is known as :

(a) Euler's equation (b) Clairant's equation

(c) Lagrange's equation (d) Cauchy's equation.

4. The orthogonal trajectories of one-parameter family $x^2 + 2y^2 = c^2$ are given by :

(a) $y = ax$ (b) $y^2 = ax$ (c) $y = ax^2$ (d) $y^2 = ax^2$.

5. The value of y given by $\frac{d^2y}{dx^2} - 3x\frac{dy}{dx} - 3y = 0$ given that $y = 1$, $\frac{dy}{dx} = 0$ where $x = 0$, is :

(a) $\sum_{n=0}^{\infty} \frac{3^n}{n!\,2^n} x^{2n}$ (b) $\sum_{n=0}^{\infty} \frac{3^n}{n}$

(c) $\sum_{n=0}^{\infty} \frac{3^n}{x^{2n}}$ (d) None of these.

6. If n is odd then $P_n(0)$ is equal to :

(a) 1 (b) $(-1)^{n/2}\frac{1.3.5 \ldots (n-1)}{2.4 \ldots n}$

(c) 0 (d) $\frac{1}{2}(n-1)$.

7. The solution of partial differential equation $r = bx$ is given by $z =$

(a) x^3 (b) $x^3 + xf(y)$

(c) $x^3 + xf(y) + g(y)$ (d) None of these.

8. If $L\{f(t)\} = f(p)$, then $L\{F(at)\}$ is equal to :

(a) $f(pa)$ (b) $\frac{1}{a}f\left(\frac{p}{a}\right)$ (c) $af\left(\frac{p}{a}\right)$ (d) $af(p)$.

9. The cosine transform of e^{-x} is :

(a) $\frac{1}{1+p^2}$ (b) $\frac{p}{1+p^2}$ (c) $\frac{1}{p^2}$ (d) $\frac{1+p^2}{p}$.

10. Inversion formula for the infinite Fourier sine transform is :

(a) $f(x) = \frac{2}{\pi}\int_0^{\infty} f(p) \sin xp\, dp$ (b) $f(x) = \frac{1}{\pi}\int_0^{\infty} f(p) \sin xp\, dp$

(c) $f(x) = \frac{2}{\pi} = \int_0^{\infty} f(p) \sin x\, dp$ (d) None of these.

Section—B $1{\cdot}5 \times 5 = 7{\cdot}5/3{\cdot}5 \times 5 = 17{\cdot}5$

1. Solve : $\sin^2 x \tan y \, dx + \sec^2 y \tan x \, dy = 0$. **Or**

Solve : $\cos x \frac{dy}{dx} + y \sin x = 1$.

2. Solve : $(a^2 - 2xy - y^2)\, dx - (x + y)^2\, dy = 0$. **Or**

Solve : $\frac{d^3y}{dx^3} - 8y = 0$.

3. Solve : $x \frac{d^2y}{dx^2} - (2x - 1) \frac{dy}{dx} + (x - 1) y = 0$. **Or**

Solve : $yz \frac{\partial z}{\partial x} + zx \frac{\partial z}{\partial y} = xy$.

4. Find : L $(\sin at)$. **Or**

Obtain : $L^{-1} \left\{ \frac{3p}{p^2 + 16} - \frac{2}{p^2 + 16} \right\}$.

5. Find the Complex Fourier transform of : $f(x) = \begin{cases} 1, & |x| < a \\ 0, & |x| > a \end{cases}$. **Or**

Find : $F_C^{-1} \{e^{-\pi p}\}$.

Section—C $6 \times 3 = 18/12{\cdot}5 \times 3 = 37{\cdot}5$

1. Solve : $x^2 \frac{d^2y}{dx^2} - 2y = x^2 + \frac{1}{x}$.
2. Solve using method of variation of parameters :

$$x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x$$

3. Show that when n is a positive integer $\pi J_n = \int_0^{\pi} \cos(n\theta - x \sin\theta)\, d\theta$.
4. Solve : $r = a^2 t$, where symbols are used in usual sense.
5. Determine the displacements $y(x, t)$ in a horizontal string stretched from the origin to the point $(\pi, 0)$ when the motion is due to the weight of the string alone. The string may be taken to be initially at rest in the position $y = 0$.