STATISTICS

Paper I : Statistical Inference

Note : Attempt **all** questions from **Section-A** (Objective type) and **Section-B** (Short answer type) and **three** questions from **Section-C** (Long/Essay type).

Section—A

$1 \times 10 = 10$

1. For the equality of two menas in case of large samples which test is used ?

(a) t-test (b) F-test (c) z-test (d) None of these.

2. Which one of the following specifies the second type of error in hypothesis testing ?

(a) Reject H_0 when it is true (b) Reject H_0 when it is false

(c) Do not reject H_0 when it is true (d) Do not reject H_0 when it is false.

3. For testing the equality of two variances, the following test is used :

(a) t-test (b) χ^2-test (c) F-test (d) Normal test.

4. For testing independence of two attributes in a $(m \times n)$ contingency table, the degree of freedom of χ^2 is :

(a) $(m + n - 1)$ (b) $(m - 1)(n - 1)$ (c) $(mn - 1)$ (d) $(m - n)$.

5. An estimator T_1 is more efficient than T_2, if :

(a) $var(T_1) = var(T_2)$ (b) $var(T_1) < var(T_2)$

(c) $var(T_1) > var(T_2)$ (d) $var(T_1) = - var(T_2)$.

6. A sufficient estimator for θ in the distribution $f(x, \theta) = \frac{1}{\theta}, 0 \le x \le \theta$ based on a random sample $X_1, X_2, \ldots, X_n$ is :

(a) $\overline{X}$ (b) $\sqrt{\Sigma X_i^2}$

(c) $\min (X_1, X_2, \ldots, X_n)$ (d) $\max (X_1, X_2, \ldots, X_n)$.

7. Statistic T is said to be an unbiased estimator of parameter θ if :

(a) $E(T) = 1/\theta$ (b) $E(T) = \theta^2$ (c) $E(T) = \theta$ (d) $E(T^2) = \theta$.

8. To obtain the confidence interval for the variance use is made of :

(a) t-statistic (b) χ^2-statistic (c) F-statistic (d) None of these.

9. Let T_1 and T_2 be sufficient statistics for θ, then :

(a) $T_1 \equiv T_2$ (b) T_1 is the function of T_2

(c) T_1 is not a function of T_2 (d) None of these.

10. The probability of type I error is not known as :

(a) size of the critical region (b) level of significance

(c) power of the test (d) None of these.

Section—B $2 \times 5 = 10$

1. Explain type I and type II errors. **Or**

What do you understand by the Power of the test ?

2. State Cramer-Rao inequality. **Or**

State Rao Backwell theorem.

3. Obtain the maximum likelihood estimator of β using a random sample of size $n = 10$ from the population with probability density function given by :

$$f(x, \beta) = \begin{cases} (\beta + 1)x^{\beta} & ; 0 < x < 1, \beta > -1 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

For x as 1·3, 4·5, 6·0, 0·93, 0·50, 2·38, 0·30, 2·40, – 0·58 and 2·83. **Or**

X_1, X_2 and X_3 is a random sample of size 3 from a population with mean value μ and variance σ^2.

T_1, T_2, T_3 are the estimators used to estimate mean value μ, where $T_1 = X_1 + X_2 - X_3$, $T_2 = 2X_1 + 3X_3 - 4X_2$, $T_3 = \frac{1}{3}(\lambda X_1 + X_2 + X_3)$.

(i) Are T_1 and T_2 unbiased estimators

(ii) Find the value of λ such that T_3 is unbiased estimator for μ.

4. Let X have a pdf of the form :

$$f(x, \theta) = \begin{cases} \frac{1}{\theta} e^{-x/\theta} & ; 0 < x < \infty, \theta > 0 \\ 0 & ; \quad \text{otherwise} \end{cases}$$

To test $H_0 : \theta = 2$ against $H_1 : \theta = 1$, use the random sample x_1, x_2 of size 2 and define a critical region :

$$W = \{(x_1, x_2) : 9{\cdot}5 \le x_1 + x_2\}$$

Find : (i) Power of the test, (ii) Significance level of the test. **Or**

Find an unbiased estimator of $e^{-3\lambda}$ on the basis of only one observation from a Poisson population $P(\lambda)$.

5. Write short note on Neyman-Pearson Lemma. **Or**

One observation is drawn from a population with pdf as :

$$f(x, \theta) = \frac{1}{\theta}, 0 < x \le \theta.$$

If the Critical region for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$ is $1 \le x \le 1{\cdot}5$, what is the value of α ?

Section—C $10 \times 3 = 30$

1. (a) State and prove Invariance property of consistency.

(b) A random sample of two elements is drawn with replacement from a population with unknown mean μ. The measures of these units are x_1 and x_2. The following two estimators are suggested for μ.

(i) $t_1 = \frac{x_1 + x_2}{2}$ (ii) $t_2 = \frac{3x_1 + 2x_2}{5}$

Are both these estimators unbiased ? Which of the two is more efficient?

2. Let P be the probability that a coin will fall Head in a single toss in order to test $H_0 : P = \frac{1}{2}$ against $H_1 : P = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 heads are obtained. Find the probability of type I error and Power of the test.

3. State and prove Neyman-Pearson's Lemma.

4. State and prove Cramer-Rao Inequality with its regularity aditions.

5. Write short notes on any two of the following :

(a) Interval estimation, (b) Application of χ^2-test, (c) Large sample test, (d) Uniformly most powerful test.