

MATHEMATICS

Paper I : Linear Algebra and Matrices

Note : Attempt all questions from Section-A (Objective type questions) and Section-B (Short answer type questions) and three questions from Section-C (Long/Essay type questions).

Section—A

1 × 10 = 10

1. If A and B are two matrices such that rank of A = m and rank of B = n, then :

- (a) rank (AB) = mn (b) rank (AB) ≥ rank A
 (c) rank (AB) ≥ rank B (d) rank (AB) ≤ min (rank A, rank B).

2. Which matrix is orthogonal :

- (a) $\begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}$ (b) $\begin{bmatrix} \sin \alpha & -\sin \alpha \\ \cos \alpha & \cos \alpha \end{bmatrix}$
 (c) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ (d) None of these.

3. If λ is characteristic root of a matrix A, then characteristics root of A^{-1} is :

- (a) $\frac{1}{\lambda}$ (b) λ (c) λ^2 (d) $\frac{1}{\lambda^2}$.

4. Every diagonal element of Skew Symmetric Matrix :

- (a) Unity (b) Zero (c) Non zero (d) None of these.

5. Which one is skew hermitian matrix :

- (a) $\begin{bmatrix} 1 & -1+i \\ -1+i & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1+i \\ -1+i & 0 \end{bmatrix}$
 (c) $\begin{bmatrix} 0 & -1-i \\ 1+i & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -1+i \\ 1-i & 0 \end{bmatrix}$.

6. Rank of Matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$:

- (a) 2 (b) 3 (c) 1 (d) None of these.

7. The vector space $V_3(\mathbb{R})$ is of dimension :

- (a) 1 (b) 3 (c) 2 (d) None of these.

8. Which is true in following :

- (a) $(\mathbb{C}, +, \cdot)$ is vector space over the field \mathbb{R}
 (b) $(\mathbb{R}, +, \cdot)$ is vector space over field \mathbb{C}
 (c) $(\mathbb{Q}, +, \cdot)$ is vector space over field \mathbb{Q}
 (d) Set of integer is vector space over field \mathbb{Q} .

9. Which is non-singular matrix :

(a) $\begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} \sin \alpha & \cos \alpha \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \end{bmatrix}$.

10. Inverse of matrix : $A = \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}$

(a) $\cos^2 \theta \begin{bmatrix} 1 & -\tan \theta \\ \tan \theta & 1 \end{bmatrix}$ (b) $\sec^2 \theta \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}$
 (c) $\cos^2 \theta \begin{bmatrix} 1 & \tan \theta \\ -\tan \theta & 1 \end{bmatrix}$ (d) None of these.

Section—B

2 × 5 = 10

1. Solve the equation by Matrix method :

$$x + 2y + z = 2, 2x + 6y + 5z = 4, 2x + 4y + 3z = 4.$$

2. Find the characteristic roots of Matrix :

$$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

3. Find the rank of matrix, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$.

4. Show that the set $\{(2, -1, 0), (3, 5, 1), (1, 1, 2)\}$ forms a basis of $V_3(\mathbb{R})$.

5. Write short note :

(a) Linear sum of two subspaces, (b) Direct sum of subspaces.

Section—C

3 × 15 = 45

1. (a) Find the characteristic roots of matrix and show that matrix satisfies its characteristic equation :

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

(b) Find the rank : $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & -1 \\ 1 & 1 & 5 \\ -1 & 0 & -3 \end{bmatrix}$

2. To show that there exists a basis for each finite dimensional vector space.

3. (a) Solve the equation by matrix method :

$$2x - y + 3z = 9, x + y + z = 6, x - y + z = 2.$$

(b) Show that : $W = \{(x, y, 5z) : x, y, z \in \mathbb{R}\}$ is vector subspace of $\mathbb{R}^3(\mathbb{R})$.

4. If W is a subspace of dimension m of a vector $V(\mathbb{F})$ of dimension n , then the dimension of the quotient space V/W .

5. (a) If in an inner product space $\|x + y\| = \|x\| + \|y\|$ then prove that x, y are linearly dependent.

(b) Find the dual basis of the basis set $B = \{(1, -1, 3), (0, 1, -1), (0, 3, -2)\}$ for $V_3(\mathbb{R})$.